Advanced Math
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32) $\cos^2 x = \boxed{\frac{1+\cos 2x}{2}}$

36) $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$ $\sqrt{\frac{1-\frac{12}{13}}{2}} = \sqrt{\frac{\frac{1}{13}}{2}} = \boxed{\frac{1}{\sqrt{26}}}$

38) $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sin\theta$ $\boxed{\frac{5}{13}}$

40) $\cot\frac{\theta}{2} = \frac{1}{\tan\frac{\theta}{2}}$ $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{\frac{5}{13}}{1+\frac{12}{13}} = \frac{5}{13}\cdot\frac{13}{25} = \frac{1}{5} \rightarrow = \boxed{5}$

42) $\sin\left(\frac{330^\circ}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}} = \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

$\cos\left(\frac{330^\circ}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = -\frac{\sqrt{2+\sqrt{3}}}{2}$

$\tan\left(\frac{330^\circ}{2}\right) = \frac{\sin\theta}{1+\cos\theta} = \frac{-\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{-\frac{1}{2}}{\frac{2+\sqrt{3}}{2}} = \frac{-1}{2+\sqrt{3}}$

44) $\sin\left(\frac{135^\circ}{2}\right) = \sqrt{\frac{1-\left(-\frac{1}{\sqrt{2}}\right)}{2}} = \sqrt{\frac{\frac{\sqrt{2}+1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

$\cos\left(\frac{135^\circ}{2}\right) = \sqrt{\frac{1+\left(-\frac{1}{\sqrt{2}}\right)}{2}}\ \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$

$\tan\left(\frac{135^\circ}{2}\right) = \frac{\frac{1}{\sqrt{2}}}{1+\left(-\frac{1}{\sqrt{2}}\right)} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{1}{\sqrt{2}-1}$

46) $\sin\left(\frac{\pi}{12}\right) = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

$\cos\left(\frac{\pi}{12}\right) = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$

$\tan\left(\frac{\pi}{12}\right) = \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{1}{2+\sqrt{3}}$

48) $\sin\frac{u}{2} = \frac{1}{\sqrt{5}}$ $\cos\frac{u}{2} = \frac{2}{\sqrt{5}}$ $\tan\frac{u}{2} = \frac{1}{2}$

50) $\sin\frac{u}{2} = \sqrt{\frac{\sqrt{10}+3}{2\sqrt{10}}}$ or $\sqrt{\frac{10+3\sqrt{10}}{20}}$ $\cos\frac{u}{2} = -\sqrt{\frac{\sqrt{10}-3}{2\sqrt{10}}}$ or $-\sqrt{\frac{10-3\sqrt{10}}{20}}$ $\tan\frac{u}{2} = -3-\sqrt{10}$

52) $\sin\frac{u}{2} = \frac{3}{\sqrt{14}}$ $\cos\frac{u}{2} = \frac{\sqrt{5}}{\sqrt{14}}$ $\tan\frac{u}{2} = \frac{3}{\sqrt{5}}$

54) $|\cos 2x|$

56) $-\left|\sin\left(\frac{x-1}{2}\right)\right|$

58) $\pm\sqrt{\frac{1-\cos x}{2}} = 1-\cos x$

$\frac{1-\cos x}{2} = 1-2\cos x+\cos^2 x$

$1-\cos x = 2-4\cos x+2\cos^2 x$

$0 = 1-3\cos x+2\cos^2 x$

$0 = (1-2\cos x)(1-\cos x)$

$1 = 2\cos x$ $1 = \cos x$

$\left\{\frac{\pi}{3}, \frac{5\pi}{3}, 0\right\}$

60) $0 = \frac{1-\cos x}{\sin x} - \sin x$

$\sin x = \frac{1-\cos x}{\sin x}$

$\sin^2 x = 1 - \cos x$

$1 - \cos^2 x = 1 - \cos x$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

$\cos x = 0 \quad \cos x = 1$

$\{\frac{\pi}{2}, \frac{3\pi}{2}, 0\}$

64) $3\sin 2\alpha \sin 3\alpha$

$3\left(\frac{1}{2}(\cos(2\alpha - 3\alpha) - \cos(2\alpha + 3\alpha))\right)$

$\frac{3}{2}(\cos(-\alpha) - \cos(5\alpha))$

$\frac{3}{2}[\cos\alpha - \cos 5\alpha]$

66) $\cos 2\theta \cos 4\theta = \frac{1}{2}[\cos(2\theta - 4\theta) + \cos(2\theta + 4\theta)]$

$= \frac{1}{2}[\cos(2\theta) + \cos(6\theta)]$

74) $\sin 5\theta - \sin 3\theta$

$2\cos 4\theta \sin\theta$

76) $\sin x + \sin 5x = 2\sin(3x)\cos(2x)$

82) $0 = \cos 2x - \cos 6x$

$0 = -2\sin(4x)\sin(-2x)$

$0 = 2\sin 4x \sin 2x$

$0 = \sin 4x \quad 0 = \sin 2x$

$4x = 0 + \pi n \quad 2x = 0 + \pi n$

$0 + \frac{\pi}{4}n \quad 0 + \frac{\pi}{2}n$

$\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\}$

84) $\sin^2 3x - \sin^2 x = 0$

$(\sin 3x + \sin x)(\sin 3x - \sin x)$

$(2\sin 2x \cos x)(2\cos 2x \sin x) = 0$

$\sin 2x = 0 \quad \cos x = 0 \quad \cos 2x = 0 \quad \sin x = 0$

$2x = 0 + \pi n \quad \frac{\pi}{2} + \pi n \quad 2x = \frac{\pi}{2} + \pi n \quad 0 + \pi n$

$0 + \frac{\pi}{2}n \quad \frac{\pi}{4} + \frac{\pi}{2}n$

$\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$